Calculus: Homework #1 Solutions

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Page 5, #2:

a) $y = 0.2x^3 - 2.8x^2 + 80x$ cents y(5) = 305, y(10) = 520, y(20) = 1280

b) y(5) = 305 y(5.1) = 309.682
average rate of change = Δy/Δx = (309.682 - 305)/(5.1 - 5.0) = 46.82 cents per foot

y(5) = 305 y(5.01) = 305.46982
average rate of change = Δy/Δx = (305.46982 - 3.05)/(5.01 - 5.0) = 46.982 cents per foot

y(5) = 305 y(5.001) = 305.4699820
average rate of change = Δy/Δx = (305.4699820 - 3.05)/(5.001 - 5.0) = 46.9982 cents per foot

c) 47. An instantaneous rate is a derivative.

d) y(10) = 520 y(10.001) = 520.04400120
average rate of change = Δy/Δx = (520.0.4400120 - 520.0)/(10.001 - 10.0) = 44.0011 cents per foot.

y(20) = 1280 y(20.001) = 1280.1280072
average rate of change = Δy/Δx = (1280.1280072- 1280)/(20.001 - 20.0) = 128.0072 cents per foot.

e) So buying longer boards is more expensive, even though one is buying a larger quantity of boards, because larger trees are harder to find.

Page 10, q1-q10:

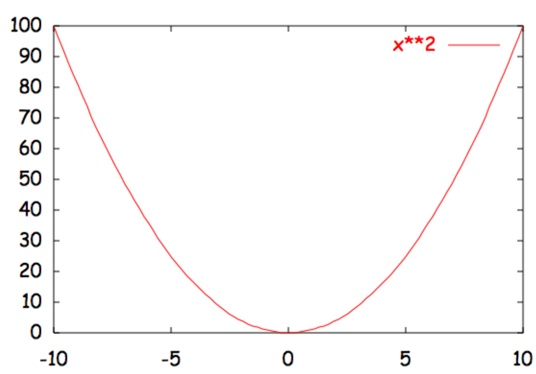
q1) cubic

q2) f(2) = 8

q3) power function

q4) g(2) = 9

q5)



q6) h(5) = 25

q7) $f(x) = ax^2 + bx + cd$

q8) y = x

q9) y = $|x|$

q10) the derivative

Page 10, #13:

a) At t=1.4 s, the table gives y=0.45 in.
With the (t=1.3, y=0.54) value, we estimate how y is changing by Δy/Δt = (0.45 - 0.54)/(1.4 - 1.3) = -0.9 in/s.
Using the (t=1.5, y=0.34) value, Δy/Δt = (0.45 - 0.34)/(1.4 - 1.5) = -1.1 in/s.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = -1.0 in/s.

At t=1.7 s, y=0.0 in.
With the (t=1.6, y=0.22) value, we estimate how y is changing by Δy/Δt = (0.0 - 0.22)/(1.7 - 1.6) = -2.2 in/s.
Using the (t=1.8, y=0.22) value, Δy/Δt = (0.0 - 0.22)/(1.7 - 1.8) = 2.2 in/s.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = 0.0 in/s.

At t=1.9 s, y=0.34 s.
With the (t=1.8, y=0.22) value, we estimate how y is changing by Δy/Δt = (0.34 - 0.22)/(1.9 - 1.8) = 1.2 in/s.
Using the (t=2.0, y=0.45) value, Δy/Δt = (0.34 - 0.45)/(1.9 - 2.0) = 1.1 in/s.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = -1.15 in/s.

b) The pebble hits the pavement when the height above the ground is zero, which occurs at t=1.2 s.

Page 10, #14:

a) At t=2 min, the table gives y=4.88 in.
With the t=0, y=6.0) value, we estimate how y is changing by Δy/Δt = (4.88 - 6.0)/(2 - 0) = -0.24 in/min.
Using the (t=4, y=4.42) value, Δy/Δt = (4.88 - 4.42)/(2 - 4) = -0.23 in/min.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = -0.235 in/min.

At t=8 min, the table gives y=3.76 in.
With the t=6, y=4.06 value, we estimate how y is changing by Δy/Δt = (3.76 - 4.06)/(8 - 6) = -0.15 in/min.
Using the (t=10, y=3.50) value, Δy/Δt = (3.76 - 3.50)/(8 - 10) = -0.14 in/min.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = -0.145 in/min.

At t=14 min, the table gives y=3.04 in.
With the t=12, y=3.26 value, we estimate how y is changing by Δy/Δt = (3.04 - 3.26)/(14 - 12) = -0.11 in/min.
Using the (t=16, y=2.84) value, Δy/Δt = (3.04 - 2.84)/(14 - 16) = -0.10 in/min.
A better estimate of the rate of change is given by averaging these two estimates: Δy/Δt = -0.105 in/min.

b) The negative sign of the rate of change means y is decreasing, that is, the tire is getting flatter.

Page 10, #15: $f(x) = x^2 + 5x + 6$ f(3) = 30.0 f(3.001) = 30.01100

$\Delta f/\Delta x$ = (30.01100 - 30.0)/ (3.001 - 3) = 11.001.

So, f is increasing at a rate of about 11.

Page 10, #16: $f(x) = -x^2 + 8x + 5$ f(1) = 12 f(1.001) = 12.005999

$\Delta f/\Delta x$ = (12.005999 - 12)/ (1.001 - 1) = 5.999.

So, f is increasing at a rate of about 6.

Page 10, #17: $f(x) = 3^x$ f(2) = 9 f(3.001) = 9.00989294

$\Delta f/\Delta x$ = (9.00989294 - 9.0)/ (2.001 - 2) = 9.89294.

So, f is increasing at a rate of about 10.

Page 10, #18: $f(x) = 2^x$ f(-3) = 0.125 f(-3.001) = 0.12491339

$\Delta f/\Delta x$ = (0.12491339 - 0.125)/ (-3.001 - (-3)) = 0.086613.

So, f is increasing at a rate of about 0.09.

Page 10, #19: $f(x) = 1/(x - 5)$ f(4) = -1 f(4.001) = -1.0010010

$\Delta f/\Delta x$ = (-1.0010010 - (-1))/ (4.001 - 4) = -1.001.

So, f is decreasing at a rate of about -1.

Page 10, #20: $f(x) = -1/x$ f(-2) = 0.5 f(-2.001) = 0.49975012

$\Delta f/\Delta x$ = (0.49975012 - 0.5)/ (-2.001 - (-2)) = 0.249875.

So, f is increasing at a rate of about 0.25.

Page 10, #21: $f(x) = -3x + 7$ f(5) = -8 f(5.001) = -8.003000

$\Delta f/\Delta x$ = (-8.00300 - 8)/ (5.001 - 5) = 3.000.

So, f is increasing at a rate of about 3.

Page 10, #22: $f(x) = 0.2x - 5$ f(8) = -3.4 f(8.001) = -3.399800

$\Delta f/\Delta x$ = (-3.399800 - (-3.4))/ (8.001 - 8) = 0.200.

So, f is increasing at a rate of about 0.2.

Page 10, #23: $f(x) = \sin x$ f(2) = 0.90929743 f(2.001) = 0.90888083

$\Delta f/\Delta x$ = (0.90888083 - 0.90929743)/ (2.001 - 2) = -0.416601.

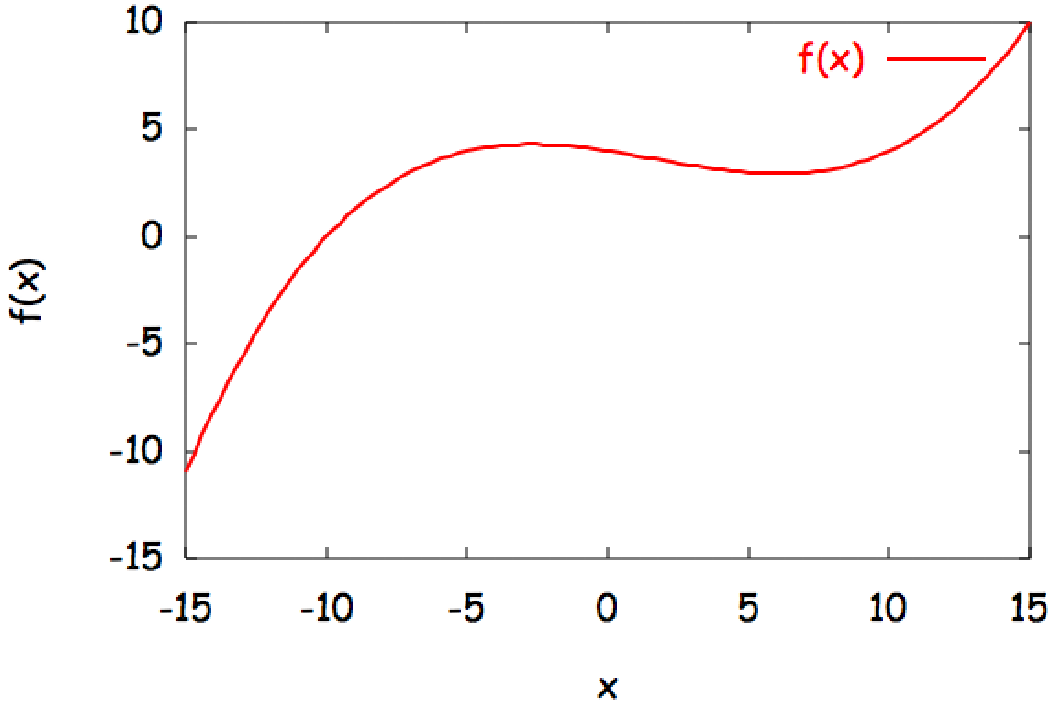
So, f is decreasing at a rate of about 0.4.

Page 10, #24: $f(x) = \cos x$ f(1) = 0.54030231 f(1.001) = 0.53946056

$\Delta f/\Delta x$ = (0.53946056 - 0.54030231)/ (1.001 - 1) = -0.841741.

So, f is decreasing at a rate of about -0.85.

Page 10, #25: $f(x) = 0.004x^3 - 0.02x^2 - 0.2x + 4$



a) f(x) is increasing quickly when the graph slopes steeply upwards.

b) f(x) is decreasing for about -3 < x < 6.

c) False. f(x) is positive but decreasing between -3 < x < 6.

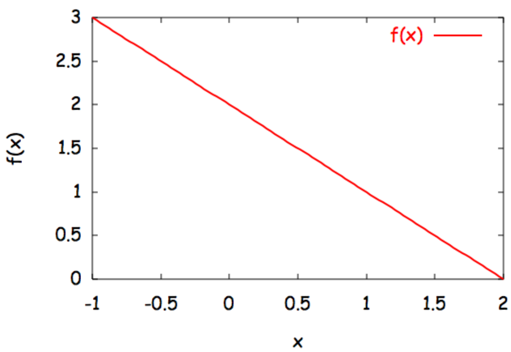
d) f(x) is decreasing the fatsest around x = 2.

Page 16, q1-q10:

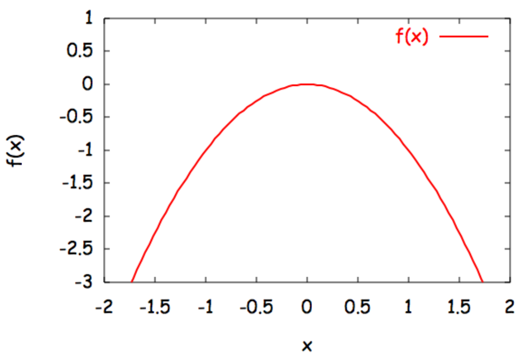
q1) f(5) = 4

q2) area = 1/2 (a+b) h = 1/2 (10+14) 6 = 12·6=72 ft^2.

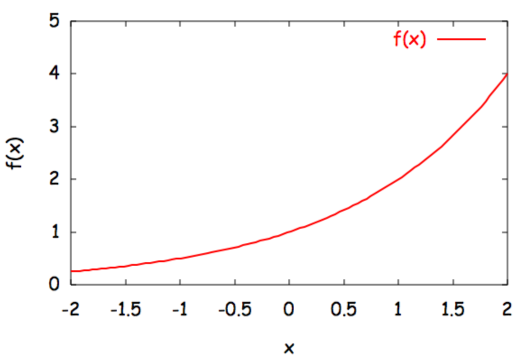
q3)



q4)



q5)



q6) x = 3

q7) y = sin(x)

q8) $y = 2^x$

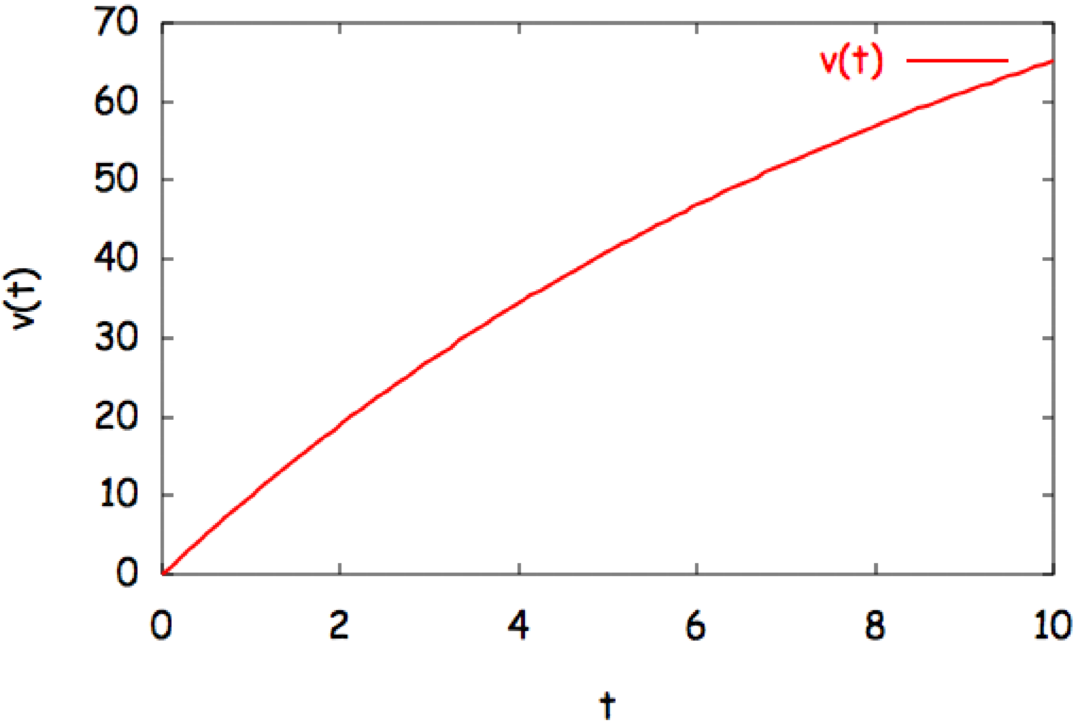
q9) y = 1/x

q10) $y = x^2$

Page 16, #6: I estimate there are 54 squares. Each square is worth 10 mi/hr · 0.5 hr = 5 mi/hr. So, the car has gone about 270 miles.

Page 16, #9:

a) $v(t) = 100\ (1 - 0.9^t)$



b) The range is $0 \leq v(t) \leq 100$

c) The speed is 60 ft/sec at about 8.7 sec

d) The car will have traveled about 1/2(60)(9) = 270 ft

e) v(t=5) = 40.951 ft/sec, and v(t=5.001) = 40.957221105.
Thus, rate is about (40.957221105 - 40.951)/(5.001 - 5.0) = 6.2 ft/sec/sec

f) We call a rate of change in the velocity an acceleration.