Calculus: Homework #2 Solutions

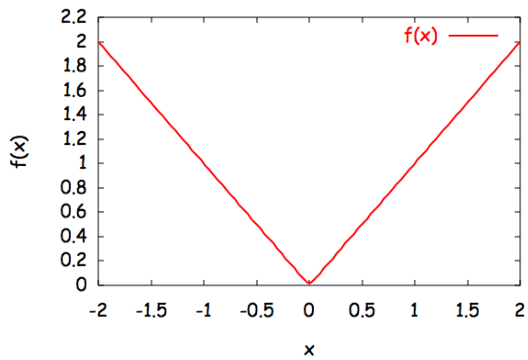
F. X. Timmes

Page 21, q1-q10:

q1) 30

q2) -500

q3)



q4) f(3) = 9

q5) 25

q6) $\sin(\pi/2) = 1$

q7) 366.25

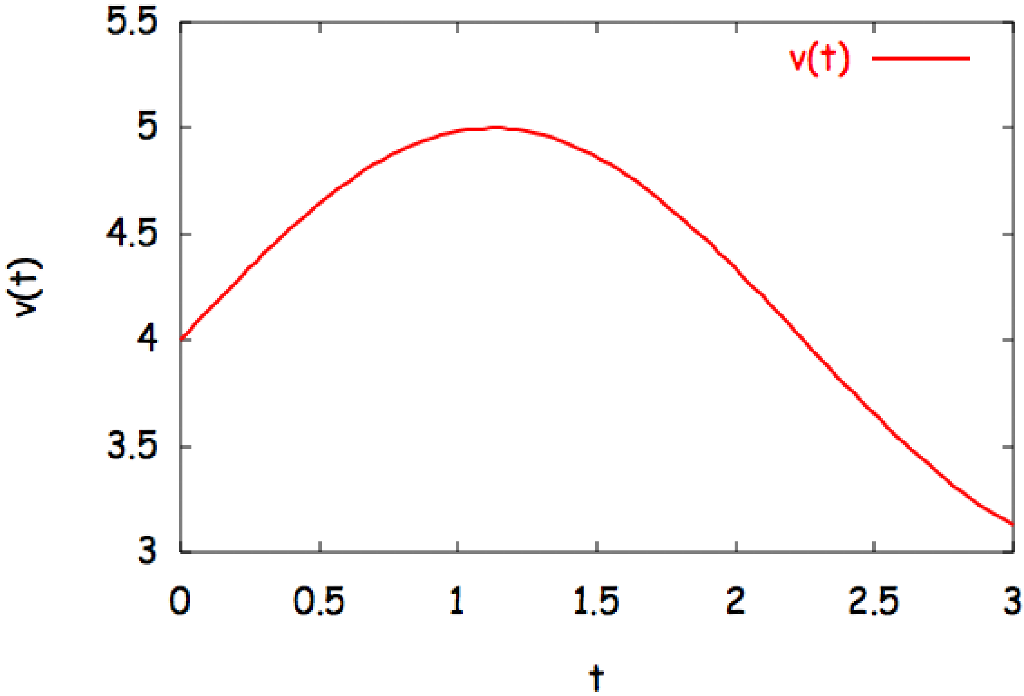
q8) derivative

q9) integral

q10) x = 4

Page 21, #2:

a) v(t) = 4 + sin 1.4t



b) While distance = speed · time, a definite integral would be used to find the distance Pace has gone in 3 hours because her speed is not constant.

c) Tabulating the function for 6 trapazoids:

t	v(t)
0.0	4.0
0.5	4.644
1.0	4.985
1.5	4.863
2.0	4.335
2.5	3.649
3.0	3.128

Using the trapezoid rule,

Integral = [1/2 · first + sum of middle + 1/2 · last] · width

Integral = [2.0 + 4.644 + 4.985 + 4.863 + 4.335 + 3.649 + 3.128]· 0.5 = 13.02 miles

Here is a fortran program to compute integrals by the trapezoid rule.

```
program trap
integer  i,n
real     lo,hi,sum,func,x,dx

lo = -1.0
hi = 6.0
n  = 100
dx = (hi - lo)/n

sum = 0.5 * func(lo)
do i=2,n
 x = lo + (i-1)*dx
 sum = sum + func(x)
enddo
sum = sum + 0.5 * func(hi)
sum = sum * dx

write(6,*) sum
end


function func(x)
real x
func = -0.1*x*x + 7.0
return
end
```

Page 21, #9: $f(x) = -0.1x^2 + 7$

a) From x=0 to x=5 in 10 increments, using the program above, I get the integral of f(x) as 30.8125

b) From x=-1 to x=6 in 10 increments, using the program above, I get the integral of f(x) as 41.7095

c) From x=-1 to x=6 in 100 increments, using the program above, I get the integral of f(x) as 41.766095

Page 21, #10: $g(x) = 2^x + 5$

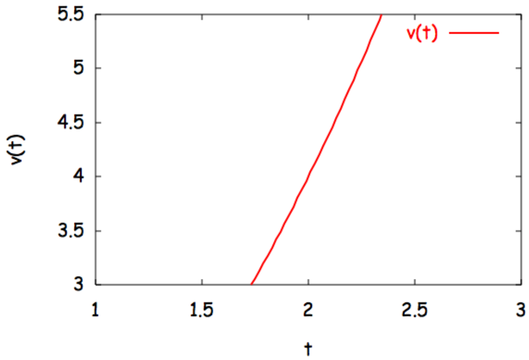
a) From x=1 to x=2 in 10 increments, using the program above, I get the integral of g(x) as 7.88654523458259

b) From x=-1 to x=1 in 10 increments, using the program above, I get the integral of g(x) as 12.1675071876618

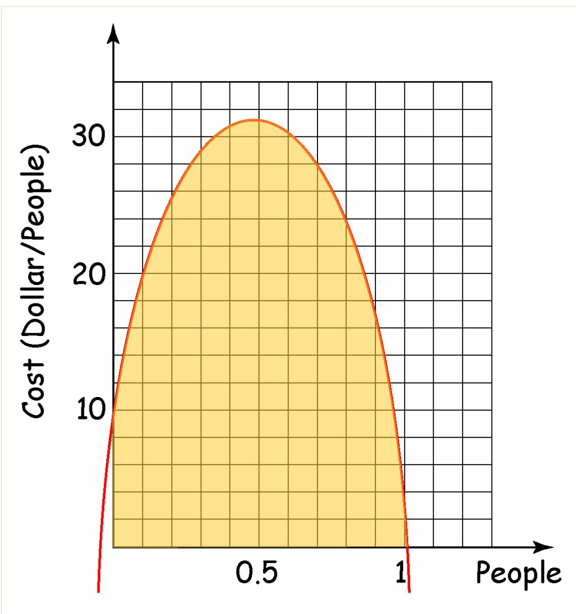
c) From x=-1 to x=1 in 100 increments, using the program above, I get the integral of g(x) as 12.1640772185815

Page 28, q1-q10:

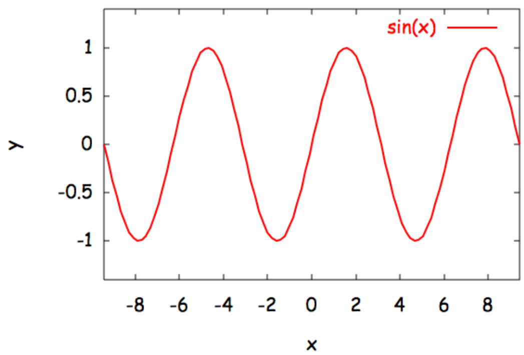
q1)



q2)



q3)



q4) $\tan(\pi/4) = 1$

q5) $1/9$

q6) $2/3$

q7) 180

q8) 52

q9) a^{-x}

q10) a derivative

Page 28, #11:

a) L is the limit of f(x) as x approaches c, if and only if, for any positive number epsilon, there is a positive number delta such that if x is within delta units of x (but not equal to c), then f(x) is within epsilon units of L.

b) f(4) = 3(4) - 7 = 12 - 7 = 5

c) L - ϵ < f(x) < L + ϵ

5 - 0.6 < 3x - 7 < 5 + 0.6

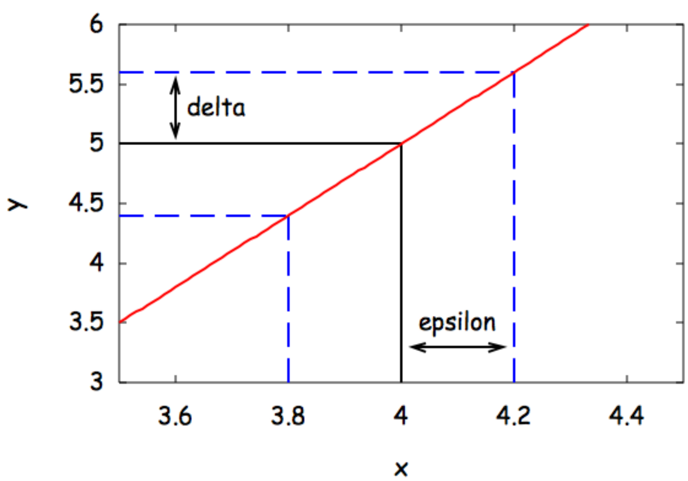
4.4 < 3x - 7 < 5.6

11.4 < 3x < 12.6

3.8 < x < 4.2

So, we must keep x within 0.2 units of 4.

d)



e) For example, let x = 4.1. Then f(4.1) = 5.3, which is within 0.6 units of 5.

e) L - ϵ < f(x) < L + ϵ

L - ϵ < 3x - 7 < L + ϵ

L - ϵ + 7 < 3x < L + ϵ + 7

L + 7 - ϵ < 3x < L + 7 + ϵ

(L + 7)/3 - ϵ/3 < x < (L + 7)/3 + ϵ/3

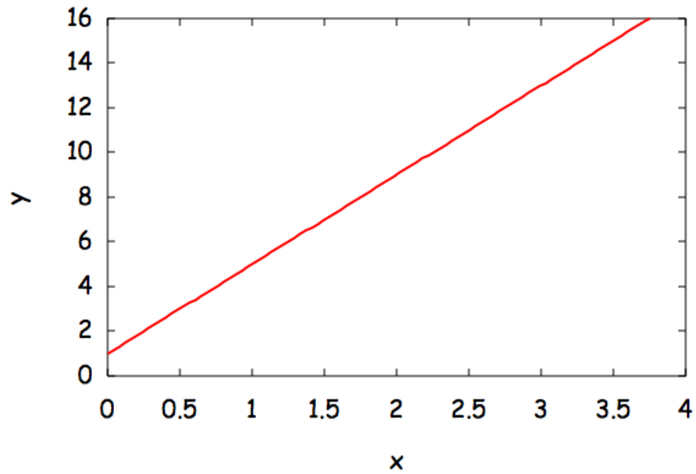
c - ϵ/3 < x < c + ϵ/3

So, we must keep x within $\delta = \epsilon/3$ of c.

If $\epsilon = 0.00012$, then we need to keep x within $\delta = \epsilon/3 = 0.00012/3 = 0.00004$ units of c.

Page 28, #12:

a) $f(x) = \dfrac{(4x^2 - 7x - 2)}{(x - 2)}$ There is a removable singularity at x = 2.



b) From the graph, the limit as x approaches 2 is 7.

c) Substituting x=2, gives f(x=0) = -2/0, which is undefined because of the division by zero.

d) $f(x) = \dfrac{(4x + 1)(x - 2)}{(x - 2)} = 4x + 1 \quad x \neq 2$

With this simplified form f(2) = 7, which is the limit as x approaches 2.

e) 8.9 < f(x) < 9.1

8.9 < 4x + 1 < 9.1

7.9 < 4x < 8.1

7.9/4 < x < 8.1/4

2 - 0.025 < x < 2 + 0.025

So, we must keep x within 0.025 units of 2.

f) L - ϵ < f(x) < L + ϵ

L - ϵ < 4x + 1 < L + ϵ

L - ϵ - 1 < 4x < L + ϵ - 1

L - 1 - ϵ < 4x < L - 1 + ϵ

(L - 1)/4 - ϵ/4 < x < (L - 1)/4 + ϵ/4

c - ϵ/4 < x < c + ϵ/4

So, we must keep x within $\delta = \epsilon/4$ of 2.

If $\epsilon = 0.001$, then we need to keep x within $\delta = \epsilon/4 = 0.001/4 = 0.000025$ units of 2.

g) c = 2, L = 7, epsilon = 0.01, delta = 0.00025.

h) I did this in part f. $\delta = \epsilon/4$.

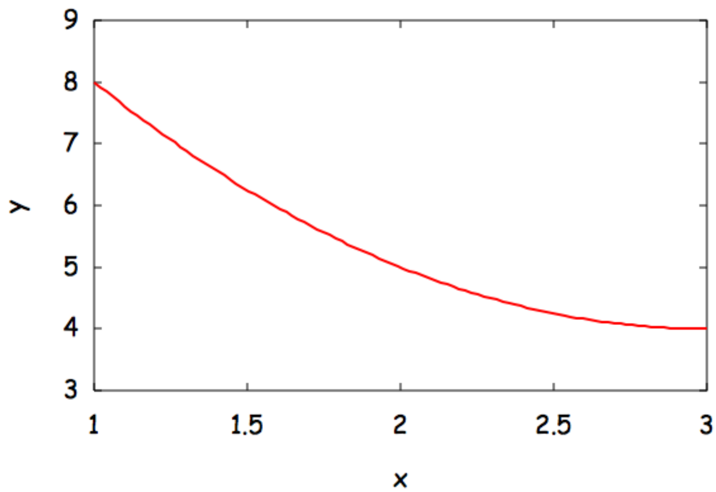
i) Without this restriction we would be allowing a divide by zero conditions in a definitions.

Page 28, #13:

a) $$f(x) = \frac{(x^2 - 6x + 13)(x-2)}{(x-2)}$$

$f(2) = \frac{0}{0}$. The limit is 5. Substitute x in to the simplified expression $f(x) = x^2 - 6x + 13, \quad x \neq 2$.

b)



x	f(x)	x	f(x)
1.990	5.020100	2.001	4.998001
1.991	5.018081	2.002	4.996004
1.992	5.016064	2.003	4.994009
1.993	5.014049	2.004	4.992016
1.994	5.012036	2.005	4.990025
1.995	5.010025	2.006	4.988036
1.996	5.008016	2.007	4.986049
1.997	5.006009	2.008	4.984064
1.998	5.004004	2.009	4.982081
1.999	5.002001	2.010	4.810000
2.000	5.000000		

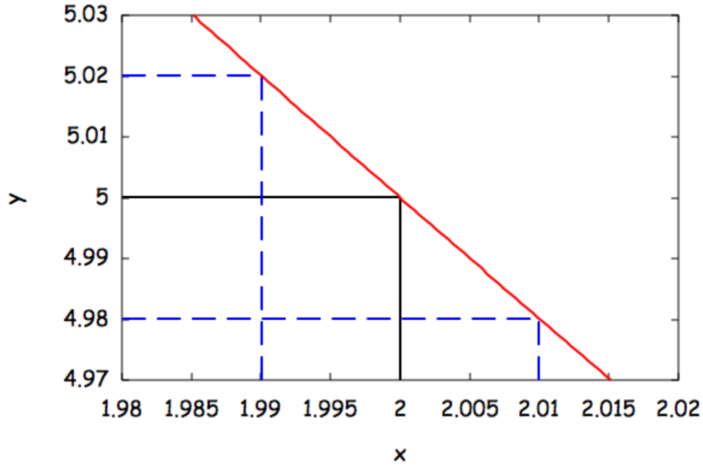
If x is within 0.004 units of 2 , then f(x) is within 0.01 units of 5. You must use the value of x that is closer to 2.

c) f(x) = 4.99: x = 2.00501256 from the quadratic equation

f(x) = 5.01: x = 1.99501243 from the quadratic equation

1.99501243 < x < 2.00501256

d)



e) From part c, we pick the smaller value of 2.0 - 1.99501243 = 0.004988 or 2.00501256 - 2.0 = 0.005013 for delta. $\delta = 0.00498756$

f) L = 5, c = 2, $\epsilon = 0.01$, $\delta = 0.00498756$