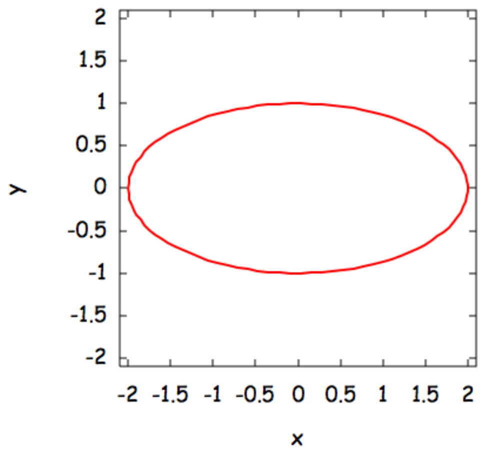
Calculus: Homework #3 Solutions

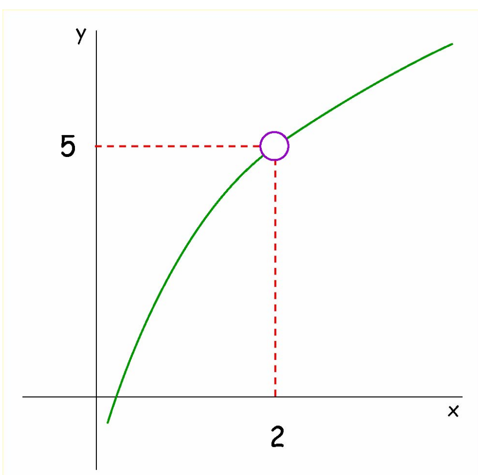
F. X. Timmes

Page 44, q1-q10:

q1)



q2)



q3) 49

q4) 5i

q5) 4

q6) plus

q7) 3

q8) Communative property of addition

q9) 200

q10) decreasing

Page 44, #8: The limit as x approaches 2 of f(x) = $(x - 2)^3 + 3$ is 3. To restrict f(x) to within ϵ=0.5 at x = 2, we have to find the value of x when f(x) = 1.5 and f(x) = 2.5.

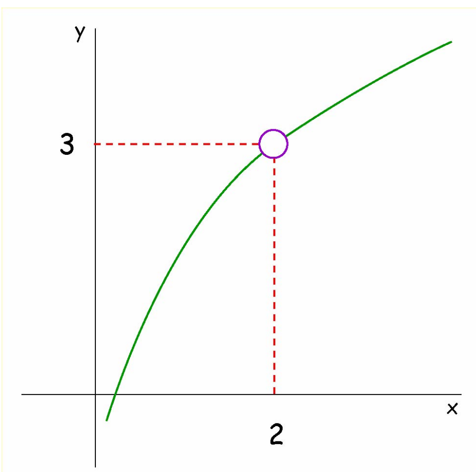
f(x) = 1.5
$(x-2)^3 + 3 = 1.5$
$(x-2)^3 = -1.5$
(x-2) = $-(1.5)^1/3$
x = 2 - $(1.5)^1/3$ = 0.855286

Similarly, for f(x) = 2.5 one finds x = 2 - $(0.5)^1/3$ = 1.206299.
Thus, delta could be 2 - 0.855286 = 1.144714 or 2 - 1.206299 = 0.793701.
We choose the smaller of these two, so δ=0.793701.

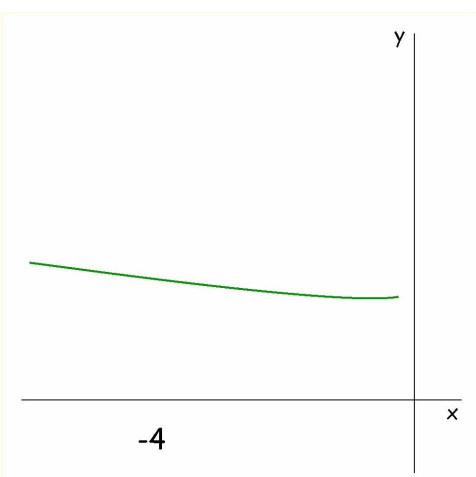
Page 49, q1-q10:

q1) 13

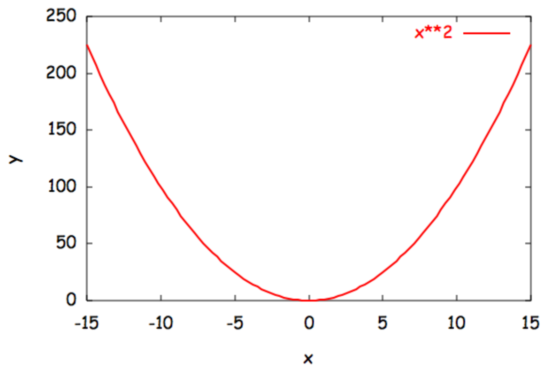
q2)



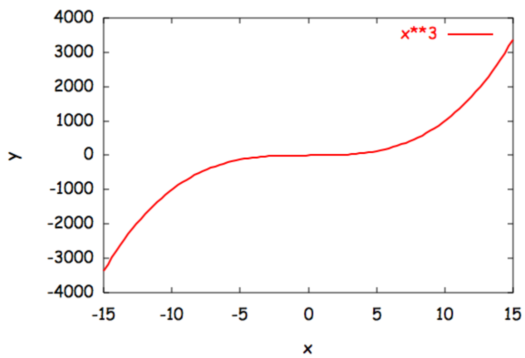
q3)



q4)



q5)



q6) (x - 10)(x + 10)

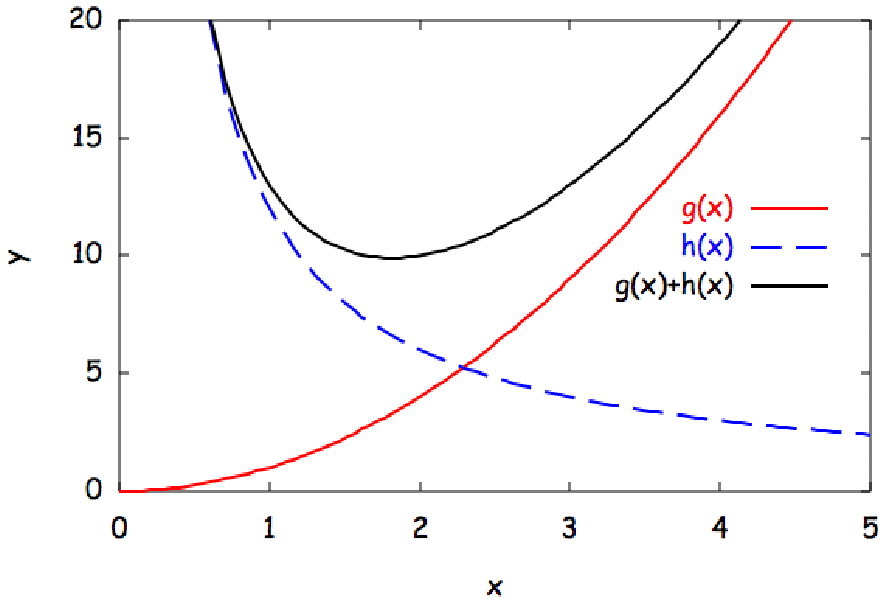
q7) 75%

q8) 10 miles

q9) $4x^2 0$

q10) The area under the curve. The product of two numbers when the numbers are changing.

Page 49, #1:



The limit as x approaches 2 of g(x) = x^2 is 4, of h(x) = 12/x = 6, and of g(x) + h(x) = 10.

x	g(x)	h(x)	g(x)+h(x)
1.995	3.980025	6.015038	9.995063
1.996	3.984016	6.012024	9.996040
1.997	3.988009	6.009014	9.997023
1.998	3.992004	6.006006	9.998010
1.999	3.996001	6.003002	9.999003
2.000	4	6	10
2.001	4.004001	5.997001	10.001002
2.002	4.008004	5.994006	10.002010
2.003	4.012009	5.991013	10.003022
2.004	4.016016	5.988024	10.004040
2.005	4.020025	5.985037	10.005062

The table shows that f(x) = g(x) + h(x) is close to the limit of 10 when x is close to 2.

Page 49, #21:

a) For f(x) = x^3, one has f(2) = 8 and f(2.1) = 9.261.

An approximation to the derivative is $\Delta f/\Delta x$ = (9.261 - 8)/(2.1 - 2.01) = 12.61.

b) We want to evaluate the quantity:

$$\frac{f(x)-f(2)}{x-2} = \frac{x^3-8}{x-2} = \frac{(x-2)(x^2+2x+4)}{x-2} = x^2+2x+4, \quad x \neq 2$$

Which approaches 12 as x approaces 2, because

$$\lim_{x\to 2} x^2+2x+4 = \lim_{x\to 2} x^2 + \lim_{x\to 2} 2x + \lim_{x\to 2} 4 = \lim_{x\to 2} x \cdot \lim_{x\to 2} x + 2\lim_{x\to 2} x + 4 = 2 \cdot +2(2) + 4 = 12$$

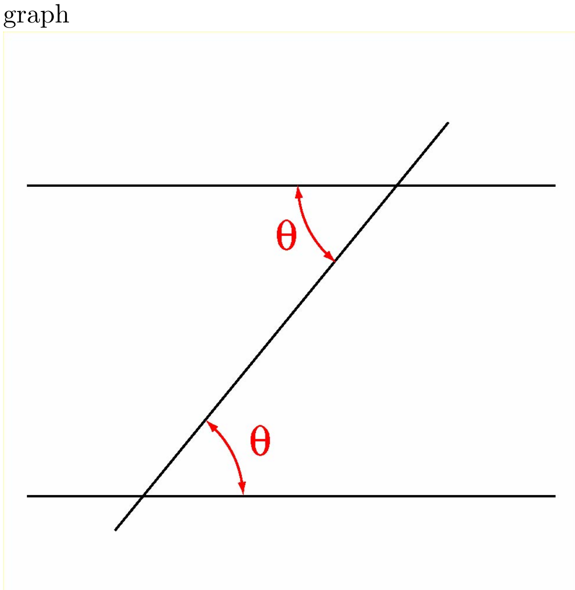
Page 56, q1-q10:

q1) Instantaneous rate of change

q2) The area under the curve. The product of two numbers when the numbers are changing.

q3) f(3) = 617. f(x) = 617.1, x = 600.1/200 = 3.005, δ = 0.005

q4) graph



q5) power function, a^x

q6) graph

q7) (x + 6)(x - 1)

q8) 53

q9) 5! = 5·4·3·2·1 = 120

q10) 28

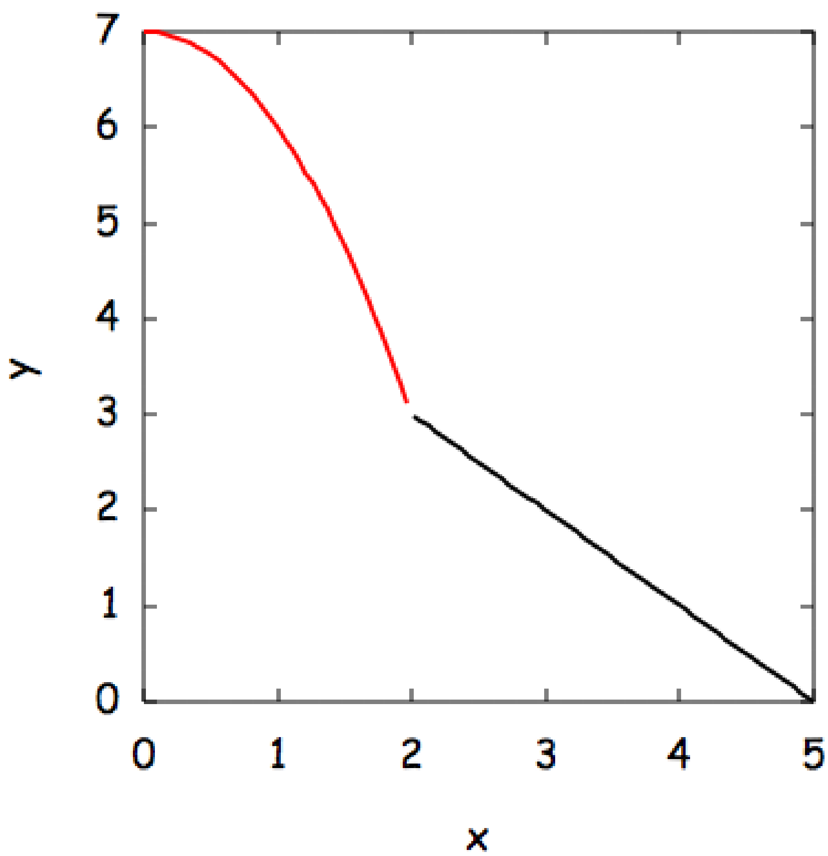
Page 56, #1: The graph has a left limit and a right limit, but they aren't equal. So, the graph doesn't have a limit and thus isn't continuous.

Page 56, #2: The graph has a left limit, a right limit, and thus has a limit. The graph isn't continuous, though, because the limit does not equal the function value.

Page 56, #3: The graph has a left limit, a right limit, and thus has a limit. The graph is also continuous, because the limit equals the function value. The rate of change is discontinuous, but not the function.

Page 56, #4: The graph has a left limit, a right limit, and thus has a limit. The graph is also continuous, because the limit equals the function value.

Page 56, #53: The function has a limit of 3 as x approaches 2 from the left, and a limit of 3 from the right. So, this function has a limit at x = 2, but is not continuous since f(2) has no value.



Page 56, #59: The function has a limit of 1.6 as x approaches 1 from the left, and a limit of 0.3 + k from the right. If we let k = 1.3, then the left and right limits would be the same, and the function would have a limit.