Calculus: Homework #4 Solutions

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Page 64, q1-q10:

q1) Does not exist. Left limit does not equal right limit.

q2) Limit exists and is equal to 3.

q3) Limit exists and equals 4.

q4) Limit exists and equals 3

q5) Limit exists and equals 2.

q6) No. Limit does not exist.

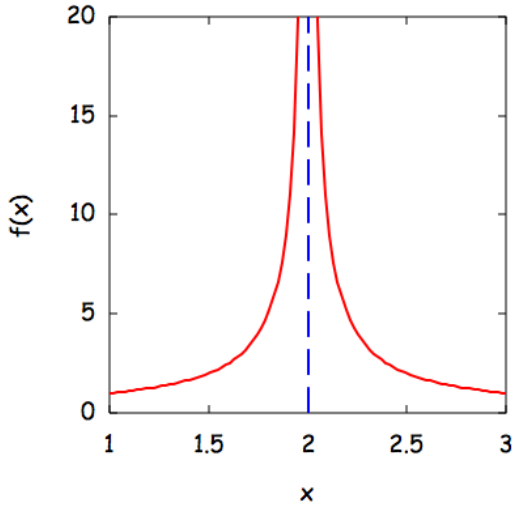
q7) No. Limit exists, but f(2) doesn't equal the limit.

q8) Yes.

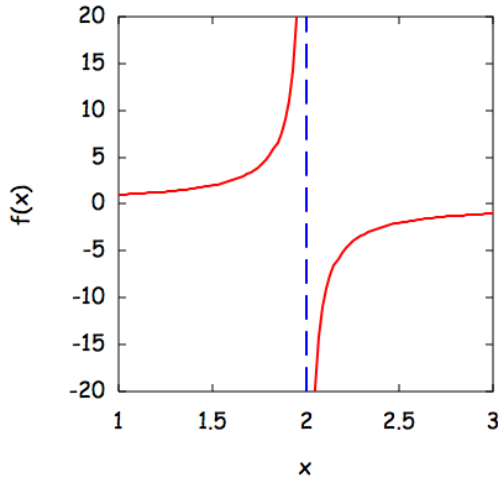
q9) No. f(4) is not defined.

q10) Yes.

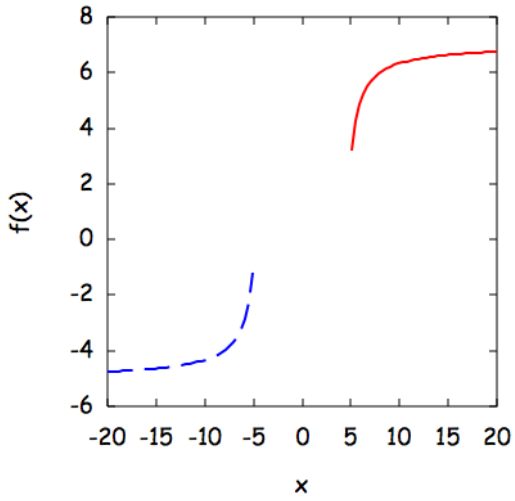
Page 64, #1:



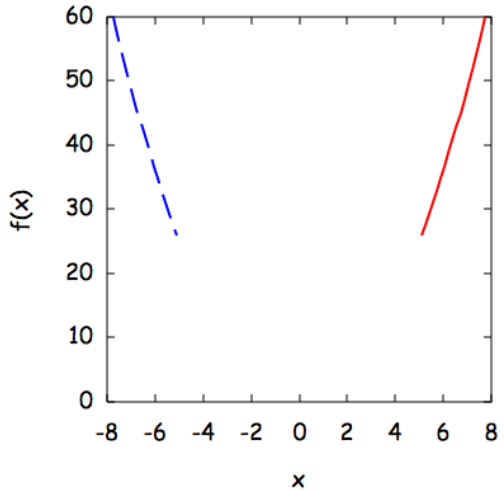
Page 64, #2:



Page 64, #3:

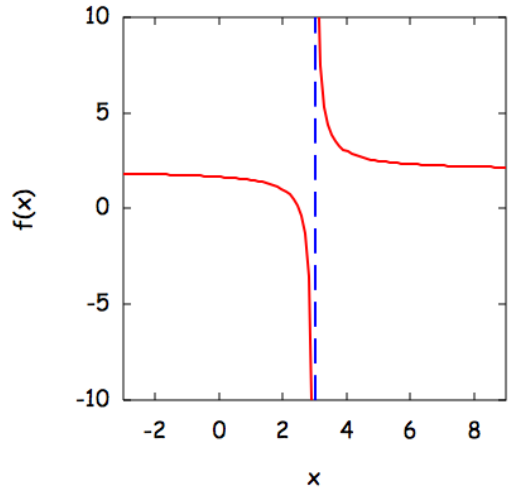


Page 64, #4:



Page 64, #5:

a. f(x) = 2 + 1/(x-3)



b. $\lim_{x\to 3^+} f(x) = \infty$ $\lim_{x\to 3^-} f(x) = -\infty$

$\lim_{x\to 3} f(x)$ = does not exist

$\lim_{x\to \infty} f(x) = 2$ $\lim_{x\to -\infty} f(x) = 2$

c. f(x) > 100 2 + 1/(x - 3) > 100 1/(x - 3) > 98 x - 3 > 1/98 x > 3 + 1/98 δ = x - 3 = 1/98

d. f(D) = 2 - 0.001 2 + 1/(D-3) = 2 - 0.001 1/(D-3) = 0.001 D - 3 = 1000 D = 1003

Page 68, q1-q10:

q1) f(2) = 3·16 + 5 = 53

q2) $\lim_{x\to 2}$ f(x) = 53

q3) h(3) is undefined

q4) $\lim_{x\to 3}$ h(x) = 5

q5) s(0) is undefined

q6) Limit does not exist. $\lim_{x\to 0^+}$ s(x) = 1 $\lim_{x\to 0^-}$ s(x) = -1

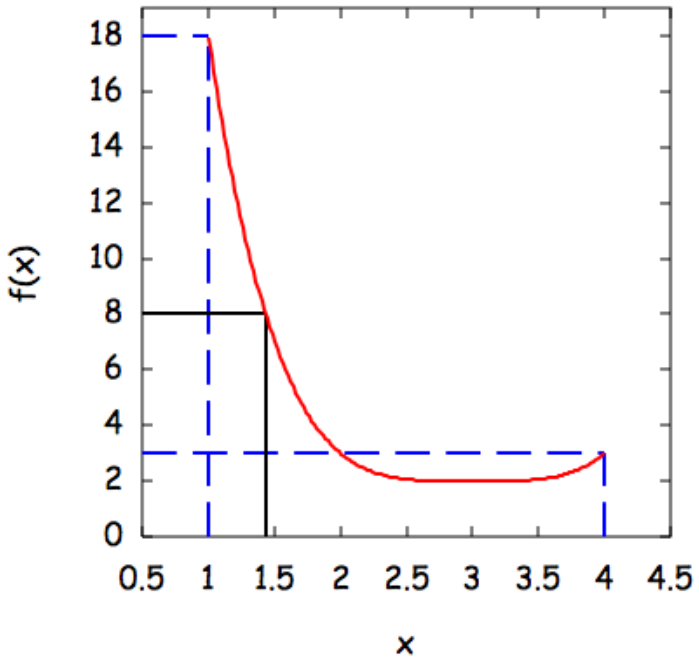
q7) sin(π/2)=1

q8) |13 - 5| = 5

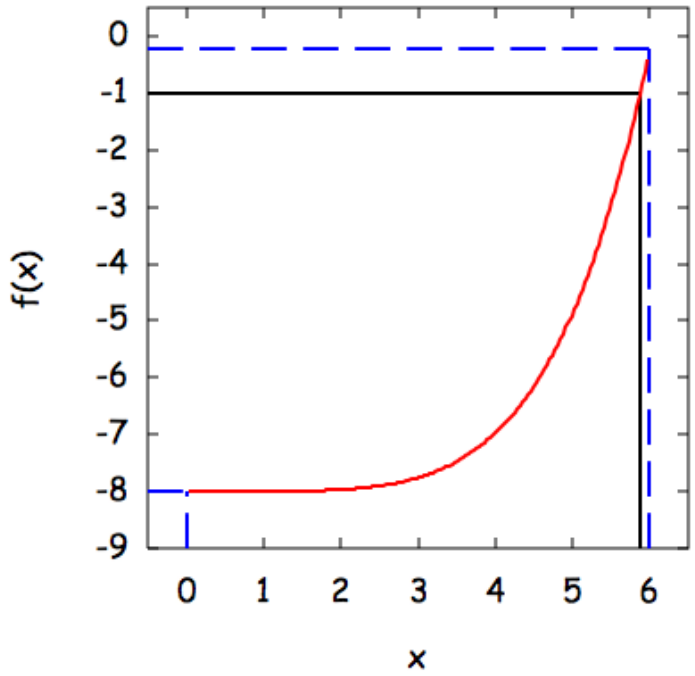
q9) log(xy) = log(x) + log(y)

q10) -3x < 12 x > 4

Page 64, #1: f(x) is a continuous function becuae it is a polynomial (limits = function values everywhere). Since f(1) = 18 and f(4) = 3, by the intermediate value theorem there is a number c between 1 and 4 such that f(c) = 8. Doing the algebra yields c = $(8 - 2)^{1/4} + 3 = 4.565085$ or 1.434915. Pick the c=1.434915 soluation since its in the [1,4] interval.



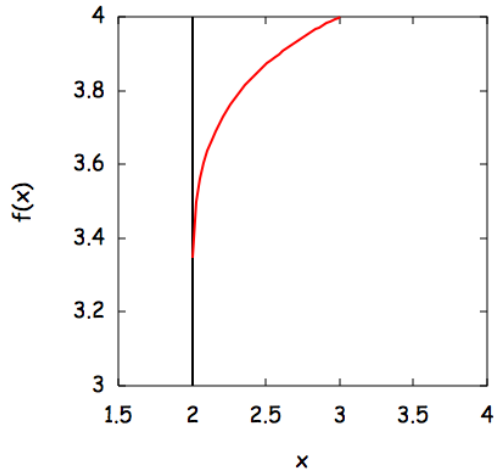
Page 68, #2: f(x) is a continuous function becuae it is a polynomial (limits = function values everywhere). Since f(0) = -8 and f(6) = -0.224, by the intermediate value theorem there is a number c between 0 and 6 such that f(c) = -1. Doing the algebra yields c = $((-1 + 8)/(0.001))^{1/5} = 5.875159$.



Page 64, #R1:

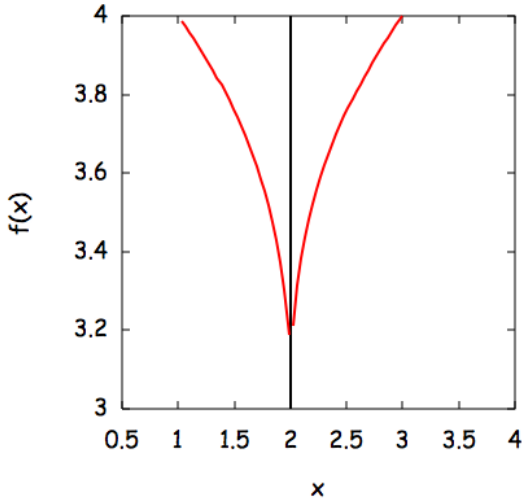
a. L = $\lim_{x \to c}$ f(x) if and only if for any $\epsilon > 0$, there is anumber $\delta > 0$ such that if x is within δ units of c, but x $\neq$ c, then f(x) is within ϵ units of L.

b. f(x) = $(x-2)^{1/5} + 3$



The graph of this function only has a branch to the right of 2. To keep f(x) within $\epsilon = 0.2$ of 3, we have to keep x within $\delta = \epsilon^5 = 0.00032$ units of 2.

c. f(x) = $(x - 2)^{2/5} + 3$



The graph of this function has a branch to the left of 2 that the function in part b did not have. The limit exists and approaches 3 as x approaches 2.