Calculus: Homework #5 Solutions

F. X. Timmes

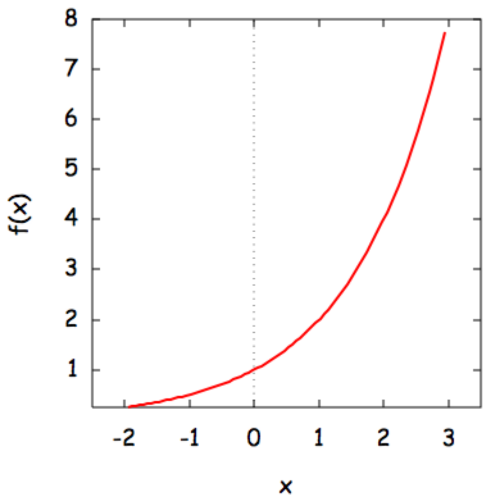
Page 81, q1-q10:

q1) Instantaneous rate of change.

q2) x + 9

q3) 18

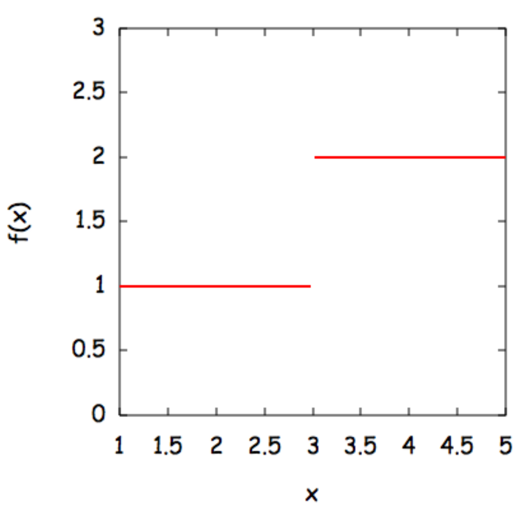
q4)



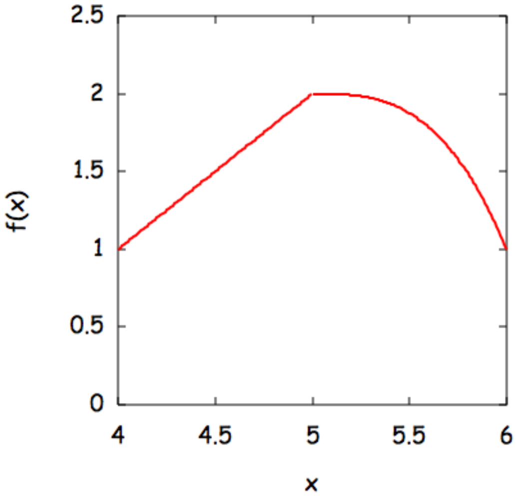
q5) $9x^2 - 42x + 49$

q6) $\log(x/y) = \log(x) - \log(y)$

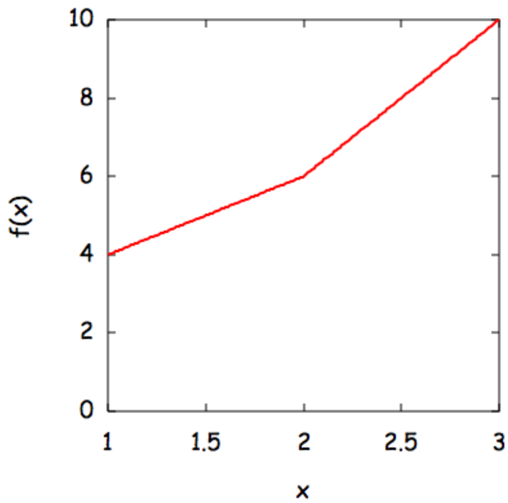
q7)



q8)



q9)



q10) Newton & Liebenitz

Page 81, #1:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Page 81, #2: Physically a derivative corresponds to a rate of change, geometrically it corresponds to the slope of a tangent line.

Page 81, #3: f(x) = $0.6x^2$, c=3

a.
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3} \frac{0.6x^2 - 5.4}{x - 3}$$

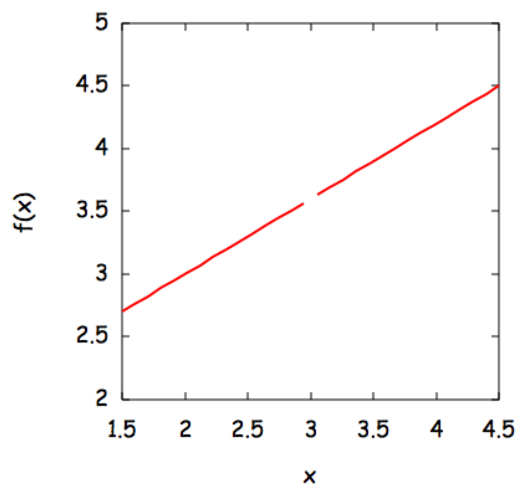
$$= \lim_{x \to 3} \frac{0.6(x^2 - 9)}{x - 3}$$

$$= \lim_{x \to 3} \frac{0.6(x + 3)(x - 3)}{x - 3}$$

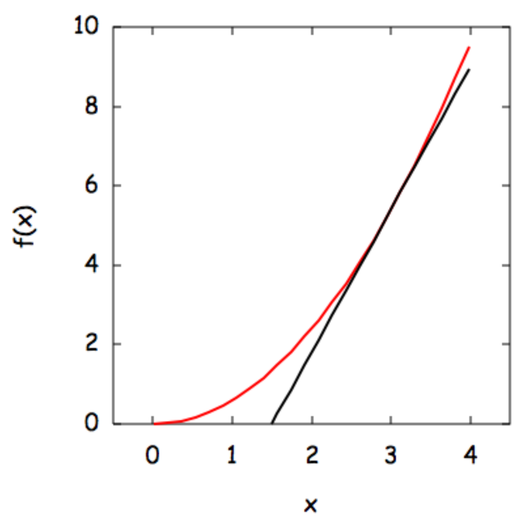
$$= \lim_{x \to 3} 0.6(x + 3)$$

$$= 0.6(3 + 3) = 3.6$$

b. The difference quotient is (f(x) - f(c))/(x - c) which in the neighborhood of c = 3 looks like:



c & d. A plot of the function and the (tangent) line through the point(c,f(c)) with slope f′(c):



Page 81, #4: f(x) = -0.2x^2, c=-4

a.

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(3) = \lim_{x \to -4} \frac{f(x) - f(-4)}{x + 4}$$

$$= \lim_{x \to -4} \frac{0.2x^2 - 3.2}{x + 4}$$

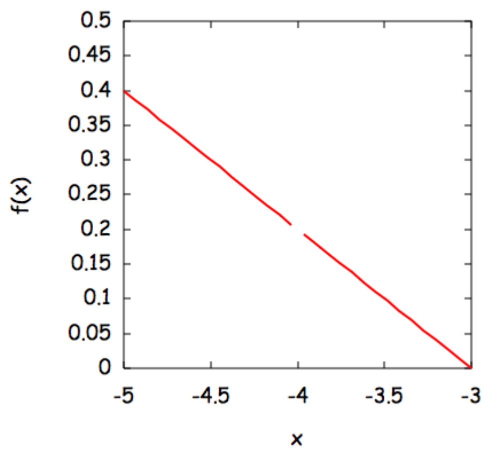
$$= \lim_{x \to -4} \frac{0.2(x^2 - 16)}{x + 4}$$

$$= \lim_{x \to -4} \frac{0.2(x + 4)(x - 4)}{x + 4}$$

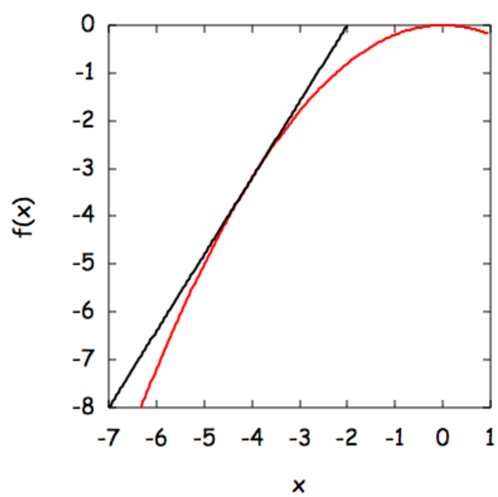
$$= \lim_{x \to -4} 0.2(x - 4)$$

$$= 0.2(-4 - 4) = -1.6$$

b. The difference quotient is (f(x) - f(c))/(x - c) which in the neighborhood of c = -4 looks like:



c & d. A plot of the function and the (tangent) line through the point(c,f(c)) with slope f′(c):



Page 81, #5: f(x) = x^2 + 5x + 1, c=-2

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x - (-2)}$$

$$= \lim_{x \to -2} \frac{x^2 + 5x + 1 - (-5)}{x + 2}$$

$$= \lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2}$$

$$= \lim_{x \to -2} \frac{(x+3)(x+2)}{x + 2}$$

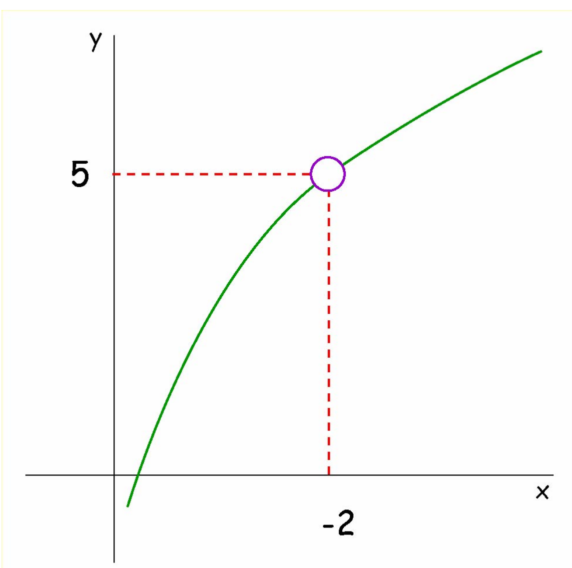
$$= \lim_{x \to -2} (x + 3)$$

$$= -2 + 3 = 1$$

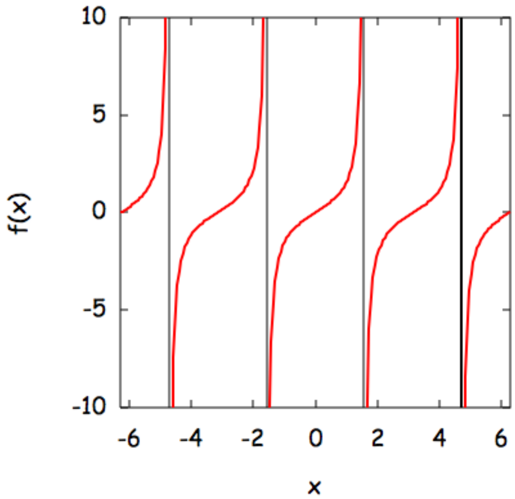
Page 87, q1-q10:

q1) 3

q2)



q3)



q4) 20%

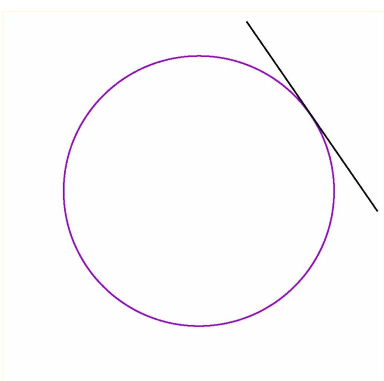
q5) $3x^2 - 2x - 8$

q6) $25x^2 - 70x + 49$

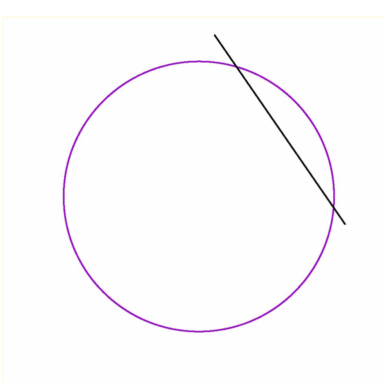
q7) $\log(36/12 \cdot 2) = \log(6)$

q8) $(x-2)\,(x^2 - 2x - 4)$

q9)

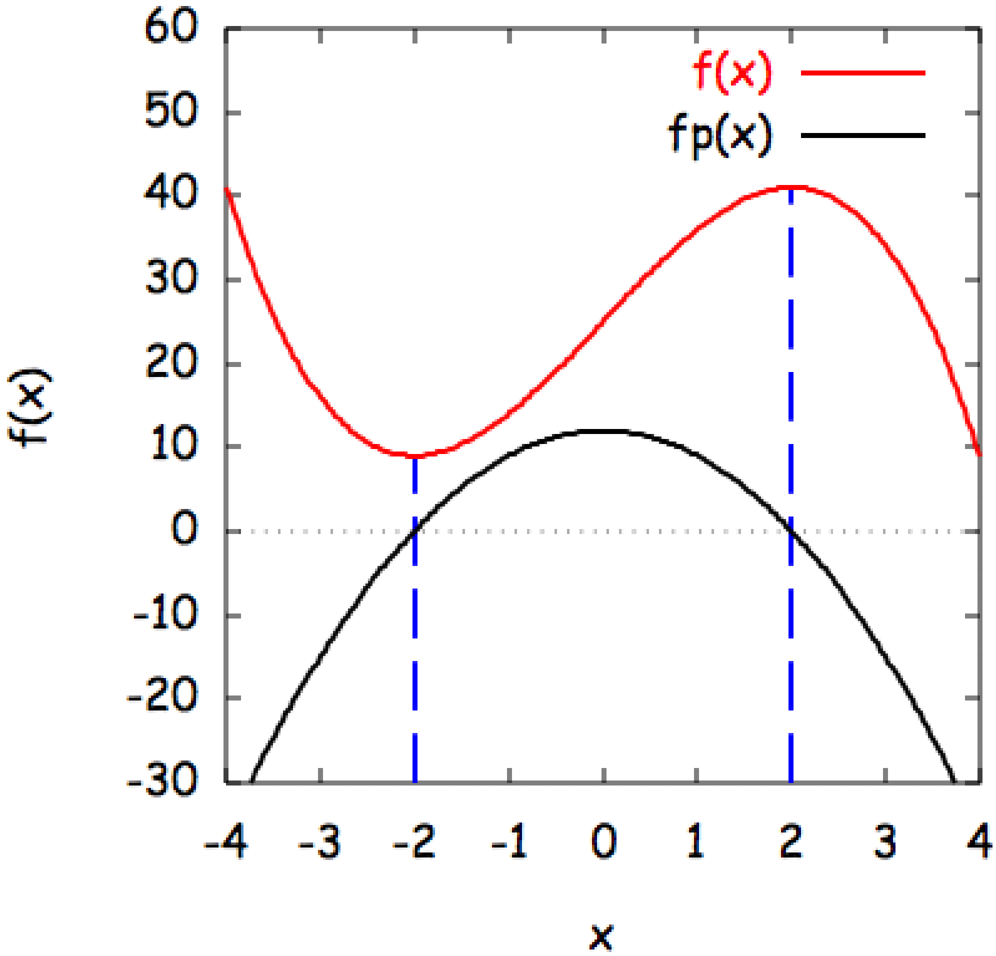


q10)



Page 87, #1: $-x^3 + 12x + 25$

a.



b. f' is positive for $-2 < x < 2$, and the graph increases in this range.

c. f(x) is decreasing for $|x| > 2$, and $f' < 0$ for all these values of x.

d. Where the f' graph crosses the x-axis, the f graph has a high point or low point.

e. See graph above.

f. Conjecture that f' is quadratic.

Page 87, #1: $f'(x) = 20x^3$

Page 87, #2: $y' = 88x^7$

Page 87, #3: $\nu' = -0.581t^{-84}$

Page 87, #4: $\nu'(x) = -1/2x^{-10}$

Page 87, #5: $M'(x) = 0$

Page 87, #6: $f'(x) = 0$

Page 87, #7: $y' = 0.6x - 8$

Page 87, #8: $r' = 0.4x + 6$

Page 87, #9: -1

Page 87, #10: $f'(x) = 9x - 1$

Page 87, #11: $y' = 2.3x^{1.3} - 10x^{-3} - 100$

Page 87, #12: $2/5x^{-3/5} - 8x + 3x^{-2}$

Page 87, #13: $\nu' = 18x - 24$

Page 87, #14: $u' = 20x - 70$

Page 87, #15: $f'(x) = 3(2x+5)^2 \cdot 2 = 6(2x+5)$

Page 87, #16: $f'(x) = 3(4x-1) \cdot 4 = 12(4x-1)$

Page 87, #17: $P'(x)) = x - 1$

Page 87, #18: $Q'(x) = x^2 + x - 1$