Calculus: Homework #11 Solutions

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Page 200, #1-6:

#1: The end point of the integration is 4, the starting point is 1, and we want 6 subintervals. This means each subinterval has a size of $\Delta x = (4 - 1)/6 = 0.5$. The midpoints of each subinterval is then given by $c_k = 1.25 + (k-1) \cdot \Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	1.25	1.5625	0.7813
2	1.75	3.0625	1.5313
3	2.25	5.0625	2.5313
4	2.75	7.5625	3.7813
5	3.25	10.5625	5.2813
6	3.75	14.0625	7.0313

The sum of the vlaues in the last column indicate that

$$\int_1^4 x^2\ dx \approx \sum_{k=1}^{6} c_k^2\ \Delta x = 20.9375$$

#2: The end point of the integration is 6, the starting point is 2, and we want 8 subintervals. This means each subinterval has a size of $\Delta x = (6 - 2)/8 = 0.5$. The midpoints of each subinterval is then given by $c_k = 2.25 + (k-1) \cdot \Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	2.25	11.3906	5.6953
2	2.75	20.7969	10.3984
3	3.25	34.3281	17.1641
4	3.75	52.7344	26.3672
5	4.25	76.7656	38.3828
6	4.75	107.1719	53.5859
7	5.25	144.7031	72.3516
8	5.75	190.1094	95.0547

The sum of the vlaues in the last column indicate that

$$\int_2^6 x^3\ dx \approx \sum_{k=1}^{8} c_k^3\ \Delta x = 319.0$$

#3: The end point of the integration is 3, the starting point is -1, and we want 8 subintervals. This means each subinterval has a size of Δx = (3 - (-1))/8 = 0.5. The midpoints of each subinterval is then given by $c_k = -0.75 + (k-1) \cdot \Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	-0.75	0.4387	0.2193
2	-0.25	0.7598	0.3799
3	0.25	1.3161	0.6580
4	0.75	2.2795	1.1398
5	1.25	3.9482	1.9741
6	1.75	6.8385	3.4193
7	2.25	11.8447	5.9223
8	2.75	20.5156	10.2578

The sum of the vlaues in the last column indicate that

$$\int_{-1}^{3} 3^x\ dx \approx \sum_{k=1}^{8} 3^{c_k}\ \Delta x = 23.9705$$

#4: The end point of the integration is 2, the starting point is -1, and we want 6 subintervals. This means each subinterval has a size of Δx = (2 - (-1))/6 = 0.5. The midpoints of each subinterval is then given by $c_k = -0.75 + (k-1) \cdot \Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	-0.75	0.5946	0.2973
2	-0.25	0.8409	0.4204
3	0.25	1.1892	0.5946
4	0.75	1.6818	0.8409
5	1.25	2.3784	1.1892
6	1.75	3.3636	1.6818

The sum of the vlaues in the last column indicate that

$$\int_{-1}^{2} 2^x\ dx \approx \sum_{k=1}^{6} 2^{c_k}\ \Delta x = 5.024$$

#5: The end point of the integration is 2, the starting point is 1, and we want 5 subintervals. This means each subinterval has a size of Δx = (2 - 1)/5 = 0.2. The midpoints of each subinterval is then given by $c_k = 1.1 + (k-1) \cdot \Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	1.10	0.8912	0.1782
2	1.30	0.9636	0.1927
3	1.50	0.9975	0.1995
4	1.70	0.9917	0.1983
5	1.90	0.9463	0.1893

The sum of the vlaues in the last column indicate that

$$\int_1^2 \sin x\ dx \approx \sum_{k=1}^{5} \sin c_k\ \Delta x = 0.95804$$

#6: The end point of the integration is 1, the starting point is 0, and we want 5 subintervals. This means each subinterval has a size of $\Delta x = (1 \text{ - } 0)/5 = 0.2$. The midpoints of each subinterval is then given by $c_k = 0.1 + (k-1)\cdot\Delta x$. A table at each interval shows

k	c_k	$f(c_k)$	$f(c_k)\ \Delta x$
1	0.10	0.9950	0.1990
2	0.30	0.9553	0.1911
3	0.50	0.8776	0.1755
4	0.70	0.7648	0.1530
5	0.90	0.6216	0.1243

The sum of the vlaues in the last column indicate that

$$\int_0^1 \cos x\ dx \approx \sum_{k=1}^{5} \cos c_k\ \Delta x = 0.8428$$

Here is the little program I used to help with the preceeding problems:

```
      program rsum

      integer  i,n
      real     lo,hi,dx,x,func,f,sum


c..user input values
c..integrate from lo to hi using n subintervals

      lo = 0.0d0
      hi = 1.0d0
      n  = 5

c..riemann sum over n midpoint intervals
      dx = (hi - lo)/float(n)
      do i=1,n
       x = lo + 0.5d0*dx + float(i-1)*dx
       f = func(x)
       sum = sum + f*dx
       print *, i,x,f,f*dx
      enddo

c..print the summation
      print *, sum
      end


c..a user defined function, here its a cosine
      function func(x)
      func = cos(x)
      return
      end
```

Page 208, q1 - q10:

q1) $1/2\ (x+2)^2$

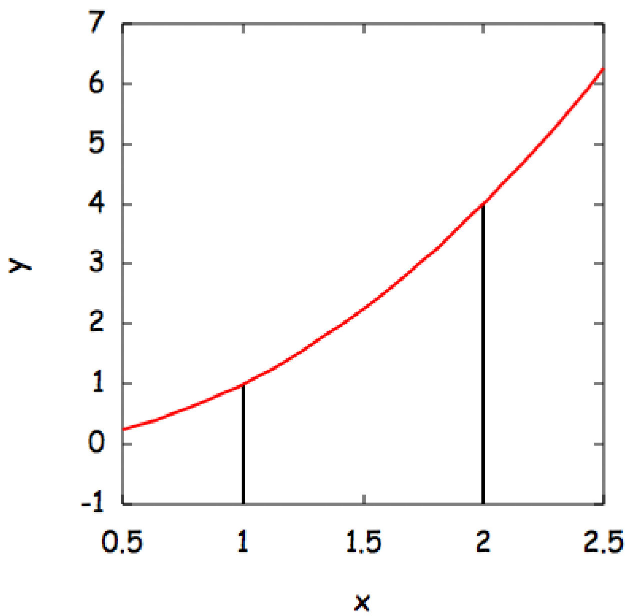
q2) $5t^2$

q3) -cot x

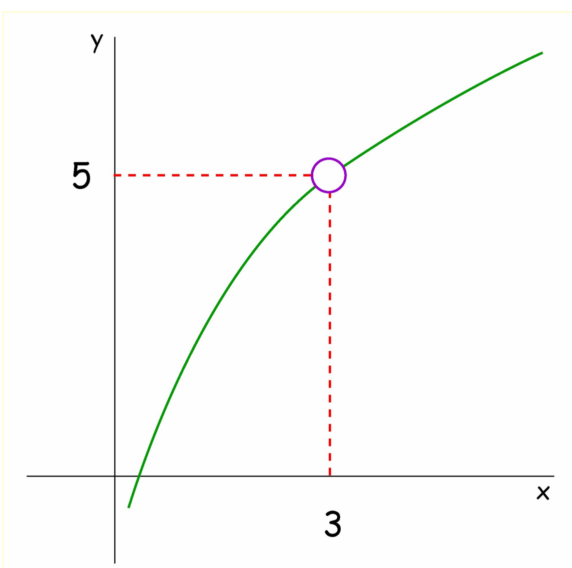
q4) -csc x tan x

q5) 5 $\sin^4$ x cos x

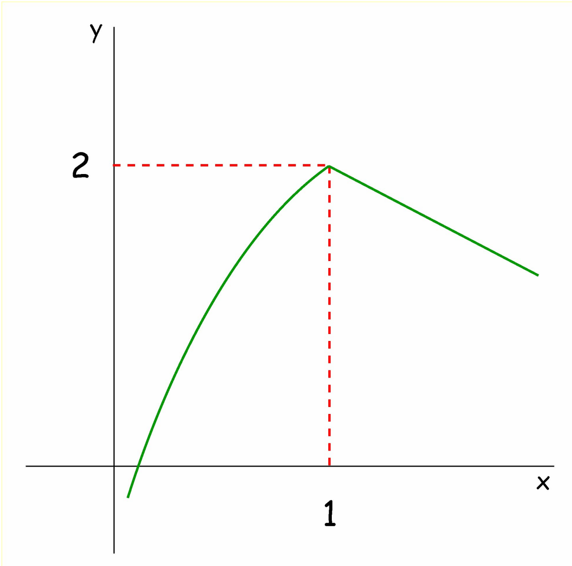
q6)



q7)



q8)



q9) ∞

q10) 0.5 x - 6

Page 208, #1: The mean value theorem says there is some point within an interval at which the instantaneous rate of the change equals the average rate of change over the whole interval.

Page 208, #2: Rolle's theorem says there is some point within an interval at which the derivative is zero if the function is zero at the endpoints of the interval. That is, there is some mountain or valley between two endpoints whose height above sea level is the same.

Page 208, #11:

a. d(50) = 1000 $(1.09)^{50}$ = \$74,357.52   No, I'm not surprised.

b. The averge rate over the 50 years is (74,357.52 - 1000)/(50 - 0) = \$1467.15/year.

c. The instantaneous rate at t=0 is about (d(0.0001) - d(0))/(0.0001) = \$86.18 /year, while at t=50 the instantaneous rate is about (d(50.0001) - d(50))/(0.0001) = \$6407.98/year. The average of these two rates is \$3247.08/year, which is certainly not what we found in part b.

d. We want to find the time where the instantaneous rate = average rate. This can be written as (d(t+0.001) - d(t))/(0.001) = \$1467.15. Or, $1.09^{t+0.001}$ - 1.09^{t} = 1.46715×10^{-3}. Playing with a few values of t, one finds t $\approx$ 33 years. This isn't halfway between 0 and 50 years.