

## Calculus: Homework #12 Solutions

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Page 217, q1 - q10:

q1)  $r' = m$

q2)  $\lim_{\Delta x \rightarrow 0} \Delta y / \Delta x$

q3) 6

q4)  $dy = \sec x \tan x \, dx$

q5)  $8x(x^2 + 3)^3$

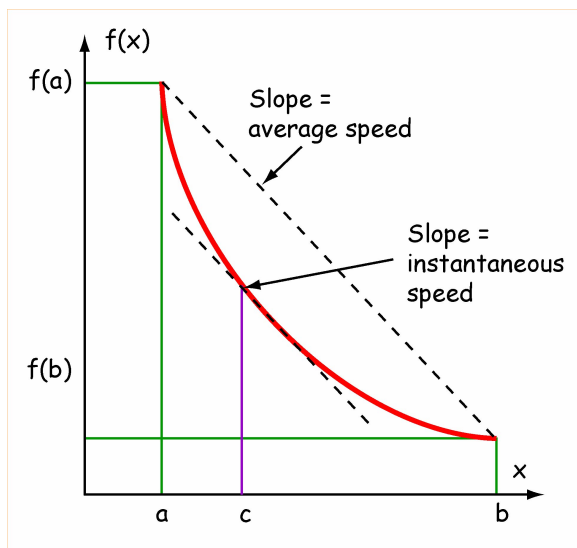
q6)  $\cos u$

q7) 0

q8)  $x = 2\pi/3$

q9) 1

q10)



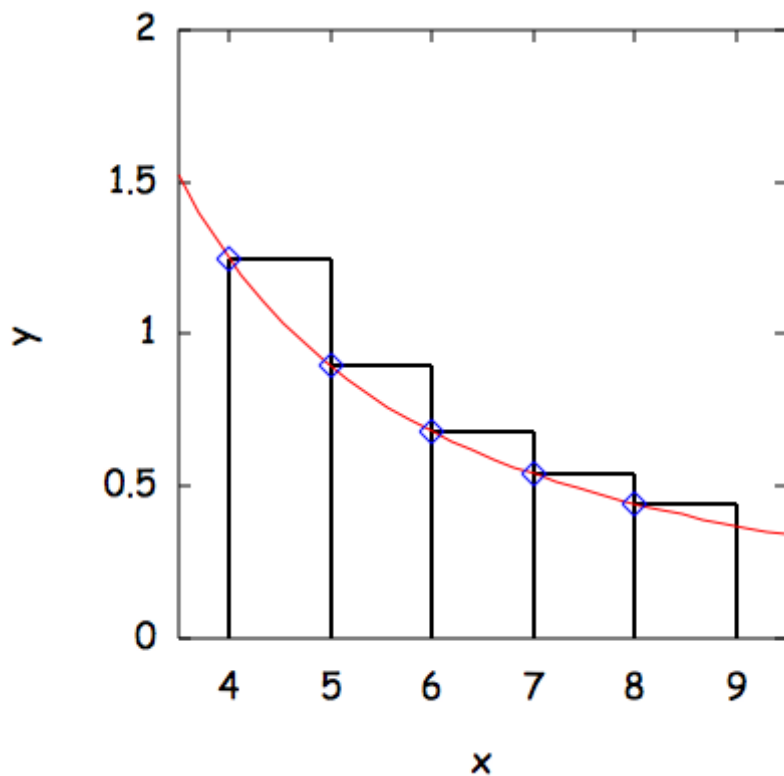
Page 217 #1:

a.

$$\int_4^9 10x^{-1.5} \, dx = -\left(\frac{10}{0.5}\right)x^{-0.5}\Big|_4^9 = -20\left(\frac{1}{\sqrt{9}} - \frac{1}{\sqrt{4}}\right) = -20(1/3 - 1/2) = 10/3$$

The “+ C” from the indefinite integral drops out when evaluating the definite integral.

b. + c. Pick the sample points at the left end of the intervals for  $U_5$ , since upper sums are to be circumscribed rectangles.



Using the program given earlier, I find  $U_5 = 3.806732 \dots$  and  $L_5 = 2.927102 \dots$ , and the average of these two to be  $3.366917 \dots$ , which overestimates the exact answer.

d. Using the midpoints, I find

$$M_{10} = 3.329112 \dots$$

$$M_{100} = 3.333291 \dots$$

$$M_{1000} = 3.333333 \dots$$

These Riemann sums seem to be converging to the exact value of the integral.

**Page 208, #2:**

a. The exact integral is

$$\int_0^{1.5} \sin x \, dx = -\cos x \Big|_0^{1.5} = -(\cos 1.5 - \cos 0) = 0.92926280 \dots$$

Using the midpoints, I find

$$M_{10} = 0.9301346 \dots$$

$$M_{100} = 0.9292715 \dots$$

$$M_{1000} = 0.9292629 \dots$$

These Riemann sums seem to be converging to the exact value of the integral.

Page 208, #3:

$$\int_a^b f(x) dx = g(b) - g(a)$$

Page 224, #1-20:

$$1. \int_1^4 x^2 dx = \frac{1}{3}x^3 \Big|_1^4 = \frac{1}{3}(4^3 - 1^3) = 21$$

$$2. \int_2^5 x^3 dx = \frac{1}{4}x^4 \Big|_2^5 = \frac{1}{4}(5^4 - 2^4) = 152 \frac{1}{4}$$

$$3. \int_{-2}^3 (1+3x)^2 dx = \frac{1}{9}(1+3x)^3 \Big|_{-2}^3 = \frac{1}{9}[10^3 - (-5)^3] = 125$$

$$4. \int_{-1}^4 (5x-2)^2 dx = \frac{1}{15}(5x-2)^3 \Big|_{-1}^4 = \frac{1}{15}(18^3 - (-7)^3) = 411 \frac{2}{3}$$

$$5. \int_1^8 60x^{2/3} dx = 36x^{5/3} \Big|_1^8 = 36(8^{5/3} - 1^{5/3}) = 1116$$

$$6. \int_1^4 24x^{3/2} dx = \frac{48}{5}x^{5/2} \Big|_1^4 = \frac{48}{5}(4^{5/2} - 1^{5/2}) = 297 \frac{3}{5}$$

$$7. \int_2^8 5 dx = 5x \Big|_2^8 = 5(8 - 2) = 30$$

$$8. \int_{20}^{50} dx = x \Big|_{20}^{50} = 50 - 20 = 30$$

$$9. \int_{-2}^0 x^2 + 3x + 7 dx = \frac{1}{3}x^3 + \frac{3}{2}x^2 + 7x \Big|_{-2}^0 = 0 - \left[ \frac{1}{3}(-2)^3 + \frac{3}{2}(-2)^2 + 7(-2) \right] = -[-8/3 + 6 - 14] = 10 \frac{2}{3}$$

$$10. \int_{-3}^0 x^2 + 4x + 10 dx = \frac{1}{3}x^3 + 2x^2 + 10x \Big|_{-3}^0 = 0 - \left[ \frac{1}{3}(-3)^3 + 2(-3)^2 + 10(-3) \right] = -[-9 + 18 - 30] = 21 \frac{2}{3}$$

11.  $\int_{-1}^1 \sqrt{4x+5} dx = \frac{1}{6}(4x+5)^{3/2} \Big|_{-1}^1 = \frac{1}{6} (9^{3/2} - 1^{3/2}) = 4 \frac{1}{3}$
12.  $\int_{-3}^3 \sqrt{2x+10} dx = \frac{1}{3}(2x+10)^{3/2} \Big|_{-3}^3 = \frac{1}{3} (16^{3/2} - 4^{3/2}) = 18 \frac{2}{3}$
13.  $\int_0^\pi 4 \sin x dx = -4 \cos x \Big|_0^\pi = -4(-1 - 1) = 8$
14.  $\int_{-\pi/2}^{\pi/2} 6 \cos x dx = 6 \sin x \Big|_{-\pi/2}^{\pi/2} = 6(1 - (-1)) = 12$
15.  $\int_{\pi/6}^{\pi/3} (\sec^2 x + \cos x) dx = \tan x + \sin x \Big|_{\pi/6}^{\pi/3} = [(\tan \pi/3 + \sin \pi/3) - (\tan \pi/6 + \sin \pi/6)] = 1.5207$
16.  $\int_0^{\pi/3} (\sec x \tan x + \sin x) dx = \sec x - \cos x \Big|_0^{\pi/3} = [(\sec \pi/3 - \cos \pi/3) - (\sec 0 - \cos 0)] = 3/2$
17.  $\int_{1.2}^{1.4} (5x+1)^6 dx = \frac{1}{35}(5x+1)^7 \Big|_{1.2}^{1.4} = \frac{1}{35}(8^7 - 7^7) = 36,388.8286$
18.  $\int_{0.4}^{0.7} (4x-1)^5 dx = \frac{1}{24}(4x-1)^6 \Big|_{0.4}^{0.7} = \frac{1}{24}(1.8^6 - 0.6^6) = 1.4152$
19.  $\int_1^2 \sin^3 x \cos x dx = \frac{1}{4} \sin^4 x \Big|_1^2 = \frac{1}{4}(\sin^4 2 - \sin^4 1) = 0.0456$
20.  $\int_{-3}^3 (1 + \cos x)^4 \sin x dx = \frac{1}{5}(1 + \cos x)^5 \Big|_{-3}^3 = \frac{1}{5} [(1 + \cos 3)^5 - (1 + \cos(-3))^5] = 0$