Shaken, Not Stirred: White Dwarf Cocktails

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Why are white dwarfs important?

The Sun will become one
Age of the universe
Probe of strong equivalence principle
Progenitors of supernovae
Element factories
Kinetic energy sources for galaxies
Records the nuclear physics of a star’s life
Probes of electron degenerate material
Sirius, only \( \sim 8.60 \pm 0.04 \) light-years from Earth, is the fifth closest stellar system. The Egyptians used Sirius, the brightest star in the sky and which rose with the Sun in early July when the Nile was in flood, to mark the first day of a New Year.
In 1844 Friedrich Bessel deduced from changes in the orbit that Sirius had an unseen companion.

In 1862, Alvan Clark first observed the faint companion during testing of the new 18.5-inch refractor telescope in the Dearborn Observatory at Northwestern University.
Strong Equivalence Principle says accelerations should be the same. Alternative theories of gravity mostly say the accelerations should be different. The outer white dwarf’s gravity accelerates the pulsar and inner white dwarf.

Microsecond accurate measurements of the radio pulse arrival times show no difference between the accelerations to 3 parts per million. Archibald et al 2018
Let’s make a white dwarf.
Modules for Experiments in Stellar Astrophysics

MESA solves the 1D fully coupled structure, mixing, and composition equations governing stellar evolution.
$L \sim 20 \, L_{\odot}$

$T \sim 2 \times 10^7 \, K$

$\rho_{(g/cc)}$

$T \sim 8000 \, K$

$1 \, M_{\odot}$

$2.0 \, M_{\odot}$

hydrogen burning

hydrogen

surface

$10^{-2}$
\[ P + P \xrightarrow{10^{10} \text{ years}} PN + e^+ + \nu \]

\[ PN + P \xrightarrow{6 \text{ sec}} PN + \gamma \text{-ray} \]

\[ PN + PN \xrightarrow{10^6 \text{ years}} NPN + P + P \]

\[ {}^{13}\text{C} \xrightarrow{(p,\gamma)} {}^{14}\text{N} \]

\[ {}^{13}\text{N} \xrightarrow{(p,\gamma)} {}^{15}\text{O} \]

\[ {}^{12}\text{C} \xrightarrow{(p,\alpha)} {}^{15}\text{N} \]
$T \sim 2 \times 10^8$ K

$\rho$ (g/cc)

helium burning

classic

to

hydrogen

surface

$T \sim 4000$ K

$L \sim 50 \, \odot$

0.1 $\odot$

0.4 $\odot$

2.0 $\odot$
\[ 3\alpha \rightarrow ^{12}\text{C} \]
\[ ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \]
\[ ^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne} \]

\[ \dot{Y}_\alpha = -\frac{1}{2}Y_\alpha^3 R_{3\alpha} - Y_\alpha Y_{12} R_{c\alpha\gamma} \]
\[ \dot{Y}_{12} = \frac{1}{6}Y_\alpha^3 R_{3\alpha} - Y_\alpha Y_{12} R_{c\alpha\gamma} \]
\[ \dot{Y}_{16} = Y_\alpha Y_{12} R_{c\alpha\gamma} \]
carbon-oxygen core

$T \sim 1 \times 10^8$ K

he-burning

hydrogen

surface $T \sim 3000$ K, $L \sim 5000$ L
An Evolution Model Aiming At KIC 08626021

$M = 0.59 \, M_\odot \quad L = 0.137 \, L_\odot \quad Teff \sim 29,000 \, K$

$\rho (10^6 \, g \, cm^{-3})$

$\rho$

$M_r/M$

$T (10^7 \, K)$

$T$

$L_r/L$

Mass Fraction

Oxygen

Carbon

Helium

$M_r/M$

Timmes et al 2018
KIC 8626021

Constellation: Cygnus

distance ~ 1000 ly
Let’s shake a white dwarf.
Like a sound wave resonating in an organ pipe, sound waves can resonate inside a star. By measuring these wave frequencies, we learn about the star’s internal structure.

Vibrations are generated by ionization and turbulence near the star’s surface. We see these oscillations as subtle, rhythmic changes in the star’s luminosity.

The vibrations penetrate into the interior, setting up resonances at frequencies dependent on density, temperature, and abundance profiles.

Resonant frequencies can vary from one every few minutes in Sun-like stars to one every few hundred days in red giants.
GYRE solves the system of equations governing small periodic perturbations to an equilibrium stellar state.
Solutions take the form

\[ \xi_r(r, \theta, \phi, t) = \text{Re} \left[ \sqrt{4\pi} \tilde{\xi}_r(r) Y^m_\ell(\theta, \phi) \exp(-i\omega t) \right] \]

\[ \xi_h(r, \theta, \phi, t) = \text{Re} \left[ \sqrt{4\pi} \tilde{\xi}_h(r) r \nabla_h Y^m_\ell(\theta, \phi) \exp(-i\omega t) \right] \]

\[ f'(r, \theta, \phi, t) = \text{Re} \left[ \sqrt{4\pi} \tilde{f}'(r) Y^m_\ell(\theta, \phi) \exp(-i\omega t) \right] \]

Solutions which satisfy the boundary conditions only occur for discrete values of the frequency \( \omega \) - these are the \textit{eigenfrequencies} of the star.
Cocktails to fit a white dwarf model’s eigenfrequencies to those derived from the Kepler mission’s photometric data:

1) Evolve a model from the main sequence to a white dwarf with the observed surface properties

2) Evolve a hot white dwarf to the observed surface properties

3) Flexible templates of the interior profiles
Template models

Oxygen Mass Fraction

Core Oxygen

Mass

Abundance

$\Delta t_1$

$\Delta t_2$

$t_1$

$t_2$

$2 \Delta t_1$

$2 \Delta t_2$

Envelope Oxygen

Giammichelle et al 2017

$\log \left(1 - \frac{M(r)}{M}\right)$
$\rho (10^6 \text{ g cm}^{-3})$

$T (10^7 \text{ K})$

$\rho_0 = 0.00 \ldots 0.25 \ldots 0.50 \ldots 0.75 \ldots 1.00$

$M_r/M_0 = 0.00 \ldots 0.25 \ldots 0.50 \ldots 0.75 \ldots 1.00$

$T_{\text{eff}} \sim 29,000 \text{ K}$

$L = 0.137 L_\odot$

Template model assumption

Evolutionary model

Lr/L

M_r/M

Timmes et al 2018
\[ \nu_{\text{relax}} - \nu_{\text{orig}} \text{ (\(\mu\)Hz)} \]

Range of Kepler observations:

\[ \ell = 1 \]

\[ \ell = 2 \]

Timmes et al 2018
We find the low order g-mode frequencies differ by up to \( \approx 70 \, \mu \text{Hz} \) over the range of Kepler observations for KIC 08626021.

By neglecting the proper thermal structure of the star (e.g., accounting for plasmon \( \nu \) losses), model frequencies calculated by assuming an \( L_r \propto M_r \) profile may have uncorrected, effectively random errors at \( \approx \) tens of \( \mu \text{Hz} \).

Extrapolating known uncertainties, a 30 \( \mu \text{Hz} \) error causes a \( \sim 12\% \) error in the white dwarf mass, a \( \sim 9\% \) error in its radius, and a \( \sim 3\% \) error in its central oxygen abundance.
Evolution of the luminosity profiles show $L_r \propto M_r$ does not occur until $T_{\text{eff}} \lesssim 20,000$ K.
Questions and Discussion
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- **log [luminosity ($L_\odot$)]**
- **log [effective temperature (K)]**

- **DOV**
- **DBV**
- **sdBV**
- **β Cep**
- **Cepheid variable**
- **δ Sct + roAp**
- **γ Dor**
- **RR Lyrae**
- **Mira**
- **Solar-like variable**

- **PVSG**
- **SPB**
- **SR**
- **RG**

- **Spectral type**
- **B**
- **A**
- **F**
- **G**
- **K**
- **M**

- **M**
- **20 $M_\odot$**
- **12 $M_\odot$**
- **7 $M_\odot$**
- **4 $M_\odot$**
- **3 $M_\odot$**
- **2 $M_\odot$**
- **1 $M_\odot$**
Critical community-driven software and data for new science.